Didactics of mathematics

Niels Grønbæk, October 2017

Most of my research life has been concerned with Banach algebras (of which operator algebras constitute the most studied class), but I have always had a strong interest in educational matters and have undergone a gradual turn so that my present interest is within Didactics of Mathematics. This field of research deals with the relationship between mathematics as a discipline and as a subject in education. Many disciplines have an associated field of didactics but Didactics of Mathematics is by far the most developed – in terms of volume, in terms of diversity of subfields. There are several reasons for this. Mathematics has a unique position in the educational system. Due to its importance as a productive power in most aspects of society all students (from primary to tertiary education) receive a substantial load and for a great many of these it is rammed down their throats or at any rate subjected to steep learning obstacles.

The radius of curvature of my turn into didactics became considerably shorter with the management driven competency debate around the 00's. The issue of competency is sound: passive knowledge alone is not enough. Knowledge becomes important only when it is in action and action stimulates the genesis of knowledge. However, to many educators in science the management imperative bastardized the core values of their disciplines. However, mathematics (in Denmark) succeeded in preserving the soundness of the concept of competency in keeping it a matter of mathematics. This is primarily the achievement of (Niss & Jensen, 2002) and witnesses the importance of a well-established field of didactics with a solid stronghold in the discipline itself.

My own contribution to clarifying the role of competency is the outcome of an invitation in 2002 from C. Winsløw (DSE, UCPH) to a development project for the course Matematik 2 AN, a first course on real analysis in the study plan, which caused extraordinary difficulties for students. Whereas (Niss & Jensen, 2002) expresses a global view on competency, we developed a theoretical local perspective, *specific competency goals* in (Grønbæk & Winsløw, 2006) and used this in the planning of course goals and student work. We further designed a format, termed *thematic projects* (Grønbæk & Winsløw, 2007), for students work, aiming at covering the mathematical core points of the course through independent student work. The key features of this format are that it allows the students to select level of sophistication and difficulty in order to optimize own ability and that it links this to a strong alignment of course goals and assessment. The thematic projects format has been the inspiration for many courses at SCIENCE, UCPH and for an assessment format in Danish upper secondary mathematics.

The phrase *didactical engineering,* put forward as a principle by the French mathematics didactician M. Artigue (Artigue, 1994), coins the close connection of didactics with lived education. This approach is behind all my work, whether it be directed towards students or teacher, separately or in union.

My second turn towards didactics was also through an invitation. C. Larsen and H. Bang, mathematics teachers from Christianshavns Gymnasium, were concerned about the quality of use of CAS (computer algebra systems) in upper secondary mathematics teaching and wanted to cooperate with MATH on the use of Maple, which was an integral part of the course Introduction to Mathematics in the mathematics study program. Eventually this led to establishing the center CMU (Danish acronym for Computer Based Mathematics Teaching) at MATH.

As pointed out by many (i.e. (Jessen, Holm, & Winsløw, 2015)) there is a severe risk that use of computers dilutes mathematical content and impoverish insight into problems of mathematical nature. This risk (contrary to a public belief) is the raison d'etre of CMU, whose effort is to ensure that the use of mathematical software in upper secondary mathematics is an enhancement solidly founded on core mathematical qualities. CMU's working mode is collaboration with teachers on designing and carrying out actual teaching with this focus.

My ongoing research is together with H. Bang and C. Larsen and concerns teachers' strategic choices on use of tools in performing (mathematical) types of activities. Any choice of tools is in dialectic relationship with the control that the (mathematical) theory, in which the activities are embodied, exerts on activity outcome. Our objective is to analyze if and how teachers deliberate their tools choices with respect to the involved mathematics control. The first analyses draw on the pool of projects completed under the auspices of CMU. In (Bang, Larsen, & Grønbæk, 2016) we outline our theoretical model, based on ATD (see below) and inspired by the business economics concepts out- and insourcing.

Such analyses involve several scientific disciplines other than mathematics: Performing mathematical tasks is a targeted human activity and as such justifiable through a discourse of its rationale. When it takes place within education this has to be understood from its institutional context. These aspects are in essence *anthropological*. What are the mental schemes at stake when one turns a Maple installation on a computer into a personal instrument and how does this influence the user's scope and use of mathematics? This is essential a problem of *cognitive ergonomics*. How are the dynamics and experience of the teacher's activity changed when digital tools mediate both teacher's and students' activity? A framework for this question is *activity theory.* How does one unfold mathematical content for students, the reverse of a research process[1], in fertile *situations* with the goal to reproduce the conditions of real mathematical activity and how does one design and deliver learning milieus for students to explore? Well, this is *mathematics* with a *strong dose of pedagogy*.

There are of course many other combinations of theories rooted outside mathematics itself with which to view mathematics in education, but these have been a basis for my work, not least because they have been developed with deep mathematical insight: Y. Chevallard founded the anthropological approach (ATD) (Chevallard, 1991) and (Chevallard, 1992); one of the first papers on instrumental approach based on cognitive ergonomics is (Guin & Trouche, 1999); activity theory appears in many connections (Monaghan, 2016); the Theory of Didactical Situations is the fundamental work of G. Brousseau (Brousseau, 1997).

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[1] So, teaching that respects this unfolding is research based.

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