My Research: Convergence and Learning of Quantum Dynamical Semigroups

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1 Mixing Times and Markovian Processes

Imagine you're playing cards with friends. Before dealing, you shuffle the deck to prevent any predictable patterns from influencing who gets what. But have you ever wondered what counts as a "good" shuffle, or how many times you need to shuffle for the deck to be truly random? Could we detect if the deck isn't well mixed, or figure out someone's secret shuffling method just by observing the cards over a few turns?

These types of questions—about mixing, randomness, and revealing hidden processes—are at the heart of my research. But instead of studying decks of cards, I focus on something we encounter less in our daily lifes: quantum systems. More specifically, I study a special type of random process known as a quantum Markovian semigroup.

More precisely, a big part of my research is devoted to properties of quantum *Markovian* semigroups. The word *Markovian* is the key here and the connection between the cards discussed above. Markovian processes are like the deck of cards being shuffled: we have a system with various possible configurations (e.g. the deck of cards and the order of the cards) and it goes through many random transitions. But crucially, the possible outcomes going from one step to the other depend only on the current state of the system. This is the Markovian property. It implies that if we know the current state of the system, it is easy to assign probabilities to the next step. However, it might be difficult to predict the behaviour after various steps. To clarify this point further, let us consider another Markovian process inspired by games: the popular snakes and ladders game depicted in Fig. 1. If we know where we are currently on the board, it is easy to compute where we could land next before we throw the die and to know the probability for each outcome. But determining the probability for where we could be after a large number of moves or how many moves are necessary to win the game on average is hard.

2 Quantum Markovian processes and what they are good for

I work on quantum systems that share this crucial property: the possible state of the quantum system after one time step only depends on the current state. And, even though analysing card and boardgames is certainly fun, the reader might be asking themseleves what this is all good for. Luckily for me and my research, it turns out that it is useful for many problems in quantum computing and mathematical physics!

Let us start with some quantum computing applications. There is currently a large effort to build quantum computers that can solve computational tasks that are beyond the capabilities of even the best supercomputers. And one of the main hurdles to achieve this goal is the fact that these systems are very *noisy*. That is, we want to send a sequence of instructions to the quantum computer, but it is unreliable: every now and then, it will not follow our instructions and do something random instead. If this happens too many times, we lose control over the state of the system and it could be in any configuration. This is akin to shuffling a deck of cards: we want to make a bunch of random moves to make sure we have no knowledge of the state of the deck. Except that for quantum computers this effect is undesirable. A significant proportion

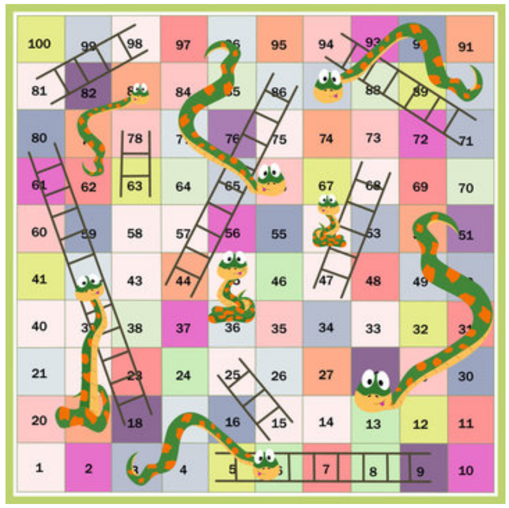

Figure 1: The snakes and ladders board game.

of my research over the last years was then to understand how different forms of noise affect the power of a quantum computer to do interesting computations from a rigorous perspective.

Benchmarking quantum devices: however, to estimate how much noise is affecting the quality of a quantum device, we first need a model of how the noise acts. And, more generally, we need to estimate with which frequency errors happen and what kinds of errors to benchmark the quality of our device and understand how to improve its quality. Answering these questions is central to another line of my work, where I develop protocols to characterize the noise affecting a quantum device efficiently.

This is a nontrivial mathematical problem for various reasons. The most daming one is the *curse of dimensionality*. To describe a general noisy process on a quantum device with n qubits[1], we need a number of parameters that scales like 16^n. Given that for current devices $n \simeq 10^2 - 10^3$, it should be clear that it is hopeless to estimate general noisy processes at such scales. Thus, it is key to identify classes of Markovian processes that have exponentially less parameters, but still capture the noise models present in true devices. And then it is important to propose protocols that can reliably estimate the parameters of such modesl from experimental data. For instance, my collaborators and I recently developed a protocol to efficiently estimate the Markovian dynamics of devices assuming it is sufficiently local by combining tools of mathematical physics and robust statistics, and an experimental team is now successfully using it to characterize systems comprised of more than 50 qubits.

Preparing quantum many-body states: another important application of quantum Markovian processes is to sample from complex probability distributions that are relevant to various physical, statistical and optimization problems. Let us again start by gaining some intuition resorting to a "real-world" situation. Let us assume we are in some large terrain and wish to find its lowest point by walking around it. One simple strategy would be to take some steps in some direction, look around and see if there is a direction going downwards from where we are. If we spot one, we walk in that direction. If there is no way to go downwards, we stay where we are and declare we found the lowest point. This is a valid strategy, but it has some clear downsides; for instance, if we encounter ourselves in some small pit, like in Fig. 2, we might think we found the lowest point, when there are even lower points ahead (we are at a local minimum). Thus, it might be worthwhile to every now and then also go in directions that will make us go up to make sure we explore more of the terrain and identify potentially lower points. So we could flip a few coins and periodically decide

[1] a qubit is the fundamental unit of quantum information, in analogy to bits in classical computers.

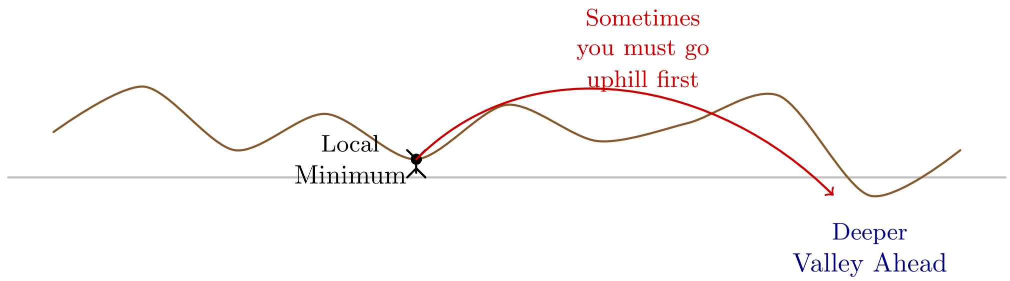

Figure 2: Illustration of local minima and why it is important to sometimes take steps in an uphill direction.

to go in a "bad direction". It turns out that a (slightly more sophisticated) version of this strategy is very effective in practice to solve a variety of problems. This class of algorithms are called Markov Chain Monte Carlo algorithms, and I work on quantum versions of such algorithms. Here, instead of finding the lowest point of a terrain, we wish to find low-energy states of a quantum systems, a fundamental problem in various areas of science such as quantum chemistry, and follow a similar recipe of random moves. I then work on developing mathematical tools to analyse the performance of such algorithms and to determine when they are effective.

The Road Ahead: In the coming years, I plan to bring these ideas together. As we push quantum devices to do ever more challenging tasks, noise will remain a bottleneck. But I suspect that some quantum Markovian algorithms are more tolerant to noise than others—and that we can design them to take advantage of how the noise behaves. Returning to the card deck analogy: if an unwanted "ace" is placed on top of the deck before a good shuffle, it ends up lost in the randomness. But place it after the shuffle, and it dominates the outcome. Learning to harness these differences could help us find reliable ways to compute, even in the presence of noise.

Ultimately, by carefully studying quantum Markovian processes—both to model noise and to engineer powerful algorithms—we stand to unlock the full potential of quantum devices, guiding them toward delivering on their remarkable promise..