# Holomorphic Day 2025

## Program, abstracts and practical information

The 31st of January, 2025

University of Copenhagen, Copenhagen, Denmark

The Conference is sponsored by Independent Research Fund Denmark | Natural Sciences via grant DFF – 1026-00267B.

## Practical information

The meeting is scheduled for Friday the 31st of January at the Department of Mathematical Sciences of the University of Copenhagen

Department of Mathematical Sciences
Universitetsparken 5
Auditorium 10
2100 Copenhagen Ø

Auditorium 10 is equipped with blackboards and a beamer.

We plan a common dinner for those that have registered. Link to registration: `https://forms.gle/XEorVKJEF6qfAf5C9` Deadline for registration is the 24th of January.

More information will be given during the day.

You may access the eduroam or KU-guest networks.

## Program

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| 09:30 – 10:00 | Coffee in the lunch room on the fourth floor of the math building |
| 10:00 – 10:50 | Marta Kosek:<br>Guided sequences of polynomials of one complex variable<br>and non-autonomous Julia sets |
| 11:00 – 11:50 | Aleksei Kulikov:<br>Contractions between Bergman spaces |
| 11:50 – 13:30 | Lunch break |
| 13:30 – 14:20 | Ana Loureiro:<br>Multiple orthogonality travelled from recurrence relations |
| 14:20 – 15:00 | Coffee |
| 15:00 – 15:50 | Nathalie Wahl:<br>From arcs in surfaces to algebraic arcs, to access symplectic groups |
| 16:00 – 16:50 | Carsten Lunde Petersen:<br>Roots of polynomial sequences in root-sparse regions |
| 17:30 – | Common dinner for registered participants |

# Abstracts

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## Guided sequences of polynomials of one complex variable and non-autonomous Julia sets

MARTA KOSEK

JAGIELLONIAN UNIVERSITY, KRAKÓW, POLAND

*K*-guided sequences of polynomials were defined in

- J. S. Christiansen, C. Henriksen, H. L. Pedersen, C. L. Petersen, *Filled Julia sets of Chebyshev polynomials*, J. Geom. Anal. 31 (2021), 12250-12263.

We discuss this notion and generalize it. Non-autonomous filled Julia sets for such sequences are defined and investigated under a natural additional assumption. We use Klimek's metric to investigate them. Just like the autonomous filled Julia sets of polynomials, they are non-empty, compact, regular with respect to the Green function and polynomially convex. Our toy example is the sequence of minimal (Chebyshev) polynomials $(t_n)_{n=1}^{\infty}$ for $[-1,1]$. This is a joint work with Małgorzata Stawiska (American Mathematical Society – Mathematical Reviews).

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## Contractions between Bergman spaces

ALEKSEI KULIKOV

UNIVERSITY OF COPENHAGEN, DENMARK

Consider spaces $A^p_\alpha$ of analytic functions $f$ in the unit disc $\mathbb{D} = \{z : |z| < 1\}$ such that $|f(z)|^p$ is integrable with respect to the probability measure $\frac{\alpha+1}{\pi}(1-|z|^2)^\alpha$ for some $0 < p < \infty$, $\alpha > -1$. It was essentially known since the work of Hardy and Littlewood which of these spaces are contained in which. In this talk we will concern ourselves with the following question: when are these embeddings contractions?

The main result that we will discuss is that if $\frac{p}{\alpha+2}$ is held constant then the $A^p_\alpha$ norms are decreasing in $p$, the proof of which notably uses isoperimetric inequality in the hyperbolic plane. We will also mention applications of these inequalities to the coefficient estimates for functions of many (and even infinitely many) variables and, if time permits, analogs of the results in other geometries.

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## Multiple orthogonality travelled from recurrence relations

ANA LOUREIRO

UNIVERSITY OF KENT, ENGLAND

Starting from a polynomial sequence satisfying a three term recurrence relation I will explain how one can describe the vector of orthogonality measures. Of course this is not always possible and restrictions have to be considered. Some somehow challenging situations will arise for us to discuss a little bit further.

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## Roots of polynomial sequences in root-sparse regions

CARSTEN LUNDE PETERSEN

UNIVERSITY OF COPENHAGEN, DENMARK

Given a family $(q_k)_k$ of polynomials, we call an open set $U$ root-sparse if the number of zeros of $q_k$ is locally uniformly bounded on $U$. We study the interplay between the individual zeros of the polynomials $q_k$ and those of the $m$th derivatives $q_k^{(m)}$, in a root-sparse open set $U$, as $k \to \infty$. More precisely, if the root distributions $\mu_k$ of $q_k$ converge weak* to some compactly supported measure $\mu$, whose potential is nowhere locally constant on a root-sparse open set $U$, then we link the roots of the $m$th derivative $q_k^m$, for an arbitrary $m > 0$, to the roots of $q_k$ and the critical points of the potential $p_\mu$ on compact subsets of $U$.

We apply this result in a polynomial dynamic setting to obtain convergence results for the roots of the $m$th derivative of iterates of a polynomial outside the filled-in Julia

set. We also apply our result in the setting of extremal polynomials. This is based on recent joint work with Christian Henriksen and Eva Uhre.

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## From arcs in surfaces to algebraic arcs, to access symplectic groups

NATHALIE WAHL

UNIVERSITY OF COPENHAGEN, DENMARK

Arcs can be used to decompose surfaces in smaller pieces and give inductive arguments, for example when studying mapping class groups of surfaces. Symplectic groups are the automorphism groups of the intersection pairing on the first homology of a surface. We will explain how algebraic "arcs" in vector spaces/modules equipped with an alternating form can be just as good as their geometric analogues, and in particular allow to prove that symplectic groups over quite general rings satisfy the same homological stability with slope 2/3 in the genus as mapping class groups of surfaces.

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