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LARGE WIDOM FACTORS

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Abstract

Let $\mathsf{E} \subset \mathbb{C}$ be an infinite compact set, and let T_n denote the nth degree Chebyshev polynomial corresponding to E. This is the unique monic polynomial of degree n which minimizes the supremum norm on E. A result by Szegő states that $\|T_n\|_{\mathsf{E}} \geq \operatorname{Cap}(\mathsf{E})^n$ for any n while Schiefermayr showed that $\|T_n\|_{\mathsf{E}} \geq 2\operatorname{Cap}(\mathsf{E})^n$ if $\mathsf{E} \subset \mathbb{R}$. Recently, Totik established that if $\mathsf{E} \subset \mathbb{R}$ then $\|T_n\|_{\mathsf{E}}/\operatorname{Cap}(\mathsf{E})^n \to 2$ if and only if E is an interval. Introducing the Widom factor

$$\mathcal{W}_n(\mathsf{E}) := \frac{\|T_n\|_{\mathsf{E}}}{\operatorname{Cap}(\mathsf{E})^n}$$

we wish to investigate the relation between the limiting behavior $\lim_n \mathcal{W}_n(\mathsf{E}) = 2$ and sets of minimal capacity, so-called Chebotarev sets. A deeper understanding of the asymptotic behavior of Widom factors have applications to Krylov subspace methods which are used in numerical linear algebra to analyze the spectral properties of matrices and solve linear systems.

Bibliography

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